Science Dynamics Masterclass
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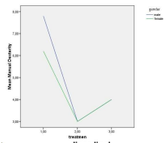
Research Labs, Proverbs.

13 - Two-way Analysis of Variance

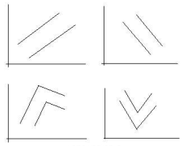
Two Factor Design

- X*Y factorial design
- SPSS individual Means
  - Row Means
    - Row Means
      also called row marginal means
- More factors allows to explain more of X units of residual variability left unexplained by one-way design

Factors Interaction



- disordinal interaction
  - disordinal...
    because the lines cross. If they do not cross (IN THE GRAPH) they are called ordinal interaction



- if one factor is constant across the level of other factor
  - if one factor is...
    then there's no interaction

Assumptions

- scores are independent
- all within cells samples come from normally distributed samples with equal variance
- the design is balanced (orthogonal)
  - the design is...
    if the number of subjects in each cell is equal

Two-wayANOVA computation

$$SS_R = \sum_{i=1}^{r} {}_RN_i\,({}_R\bar{X}_i - \bar{\bar{X}}^2)$$

Sum of squares associated with the row effect

- Sum of squares associated with the row effect
  r represents the number of levels of the row factor, rNi is the number of values in row i, rXi is the mena of the values in row i, X is overall mean of all values in the sample

$$SS_C = \sum_{j=1}^{c} {}_cN_j\,({}_c\bar{X}_j - \bar{\bar{X}}^2)$$

Sum of squares associated with the column effect

- Sum of squares associated with the column effect
  c represents number of levels of the column factor, cNj is the number of values in column j, cXj is the mean of the values in column j, X is overall mean of all values in the sample.

$$SS_{R\times C} = \sum_{i=1}^{r}\sum_{j=1}^{c} N_{i,j}\,(\bar{X}_{i,j} - {}_R\bar{X}_i - {}_c\bar{X}_j + \bar{\bar{X}})^2$$

Sum of squares associated with the interaction effect

$$SS_W = \sum_{i,j,k} (X_{i,j,k} - \bar{X}_{i,j})^2$$

Sum of squares associated with the remaining within-cell variation

$$SS_T = \sum (X_{i,j,k} - \bar{\bar{X}})^2$$

Sum of squares associated with the total variability

- Sum of squares associated with the total variability
  Xijk is the k-th score in row i and column j and X is overall mean of all values in the sample.

- within-cell values
  - dfw = N-R*C
    - Rows
    - Columns
  - MSw=SSw/dfw
- rows values
  - dfR=R-1
  - MSr=SSr/dfR
  - F=MSr/MSw
- column values
  - dfc=C-1
  - MSc=SSc/dfc
  - F=MSc/MSw
- interaction values
  - dfRxC=(R-1)(C-1)
  - MSRxC=SSRxC/dfRxC
  - MSRxC/MSw

Two-WAYANOVAnext steps

Modeling each score as a sum of Component Parts

- each score in ANOVAcomprises sum of five parts
  - grand mean
  - effect due to row factor
  - effect due to column factor
  - effect due to interaction of rows and columns
  - within-cell deviations - leftover, unexplained individual difference

$$X_{ijk} = \bar{X} + ({}_R\bar{X}_i - \bar{X}) + ({}_c\bar{X}_j - \bar{X}) + (\bar{X}_{ij} - {}_R\bar{X}_i - {}_c\bar{X}_j + \bar{X}) + (X_{ijk} - \bar{X}_{ij})$$

Measuring Effect Size

$$R^2_{ROW} = \frac{SS_{ROW}}{SS_T}$$

$$R^2_{COL} = \frac{SS_{COL}}{SS_T}$$

$$R^2_{INTER} = \frac{SS_{INTER}}{SS_T}$$

$$\eta^2_{ROW} = \frac{SS_{ROW}}{SS_{ROW} + SS_W}$$

$$\eta^2_{COL} = \frac{SS_{COL}}{SS_{COL} + SS_W}$$

$$\eta^2_{INTER} = \frac{SS_{INTER}}{SS_{INTER} + SS_W}$$

partial eta squared

SPSSComputation

- Fixed Random
  - Only Fixed
    -fixed-effect model
  - Only Random
    -random effect model
  - mixed-effect models

Post-hoc Multiple Comparison

- Three omnibus F-tests
  - row effect
  - column effect
  - interaction
- Simple effects test
  - alpha=alphaFW/C
  - alpha=alphaFW/R
  - SPSS