Science Dynamics Masterclass
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11 - Inferences involving Mean when sigma is not known

Single Sample Design

$$H_0: \mu = c$$
$$H_0: \mu \neq c$$

t-distribution

$$t = \frac{\bar{X} - \mu}{\frac{\hat{\sigma}}{\sqrt{N}}} = \frac{\bar{X} - \mu}{\hat{\sigma}_{\bar{X}}}$$

if parent population deviates from normality N>30 should be sufficient to correct it

t-distribution
varies from sample to sample, is a function of the sample size - called the degrees of freedom, symbolized by df (or Greek letter v (ny))

Confidence interval for t-distribution

$$Upper\ limit = \bar{X} + t_c \hat{\sigma}_{\bar{X}}$$
$$Lower\ limit = \bar{X} - t_c \hat{\sigma}_{\bar{X}}$$

Confidence int...
tc = idf.t(p, df), for 95% CI tc=idf.t(0.025,df)

Effect Size

$$d = \frac{\bar{X} - \mu}{\hat{\sigma}}$$

Effect Size
sigma = Std.deviation in SPSS (first table), pop.mean = test value, mean = tested mean. The same referential values apply as in previous chapter.

Degrees of Freedom for one sample t-test

Degrees of Freedom...
number of degrees of freedom is the number of independent pieces of data used in computing. With a specified mean there are only N-1 degrees of freedom for selecting a set of N scores.

Paired Samples t-test and confidence interval

Paired Sampl...
cross-sectional design (observations are taken on the same object over time)

the t-value

$$t = \frac{\bar{D} - \mu_D}{\hat{\sigma}_{\bar{D}}}$$

the t-value
D=difference Scores (mean X1-mean X2)

Degrees of Freedom

$$df = N - 1 = Number\ of\ pairs - 1$$

Standard Error of the mean difference

$$\hat{\sigma}_{\bar{D}} = \frac{\hat{\sigma}_D}{\sqrt{N}}$$

Confidence Interval

$$Upper\ limit = \bar{D} + (t_c)(\hat{\sigma}_{\bar{D}})$$
$$Lower\ limit = \bar{D} - (t_c)(\hat{\sigma}_{\bar{D}})$$

Confidence Int...
where tc is found by IDF.T(p,df) with df=N-1

Basic Assumptions are the same as in previous cases

Effect Size for paired t-test

Effect Size for...
sigmaD = paired Samples Test-Paired Differences - Std.deviation

Two Sample Design

Noise

Noise
if SIGNAL component is affected by NOISE (e.g. S-Years of Math, N-edu aspirations, academic interests etc.) are the same for both groups, noise is small.

Standard error of the mean difference

$$\hat{\sigma}_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{(N_1 - 1)\hat{\sigma}_1^2 + (N_2 - 1)\hat{\sigma}_2^2}{N_1 + N_2 - 2}\left(\frac{1}{N_1} + \frac{1}{N_2}\right)}$$

t-statistics for the two-population design

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{(N_1 - 1)\hat{\sigma}_1^2 + (N_2 - 1)\hat{\sigma}_2^2}{N_1 + N_2 - 2}\left(\frac{1}{N_1} + \frac{1}{N_2}\right)}}$$

confidence interval

$$Upper\ limit = (\bar{X}_1 - \bar{X}_2) + (t_{crit})(\hat{\sigma}_{\bar{X}_1 - \bar{X}_2})$$
$$Lower\ limit = (\bar{X}_1 - \bar{X}_2) - (t_{crit})(\hat{\sigma}_{\bar{X}_1 - \bar{X}_2})$$

confidence interval
t crit can be found using IDF.T(p,df) with df=N1+N2-2

Assumptions of the independent samples t-test

null hypothesis is true

both parent distributions are normally distributed

both parent distributions are...
violation of normality can be ignored if the sample from nonnormal parent population is sufficiently large or if the distribution of the nonnormal parent population is approximately symmetric.

both populations have equal variances (homogenityof variances)

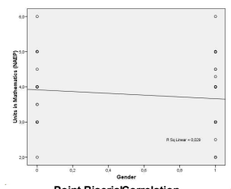
Levene'stest of variance

Levene's test...
if the p-value associated with Levene's test is less than alpha, we reject Ho. If the Leven's p-value is greater than alpha the homogenity of variance assumption is tenable.

Effect Size for Independent Samples t-test

$$d = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\frac{(N_1 - 1)\hat{\sigma}_1^2 + (N_2 - 1)\hat{\sigma}_2^2}{N_1 + N_2 - 2}}}$$

Point BiserialCorrelation



Point Biserial...
the greater magnitude the greater difference between groups

Assumption of equal variance can be avoided by using estimate of variance

$$\hat{\sigma}_S^2 = \frac{\hat{\sigma}_1^2}{N_1} + \frac{\hat{\sigma}_2^2}{N_2}$$

Degrees of Freedom in this case

$$df = v = \frac{\left(\frac{\sigma_1^2}{N_1} + \frac{\sigma_1^2}{N_2}\right)^2}{\left(\frac{1}{N_1 - 1}\right)\left(\frac{\sigma_1^2}{N_1}\right)^2 + \left(\frac{1}{N_2 - 1}\right)\left(\frac{\sigma_2^2}{N_2}\right)^2}$$

Degrees of Fr...
doesn't have to be integer in all cases!